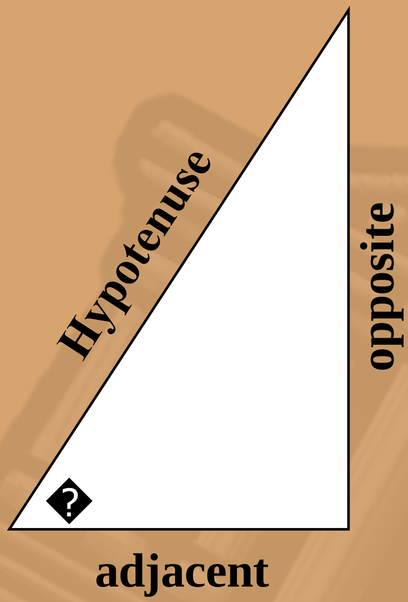
Math Lo.9

Qena Student Club

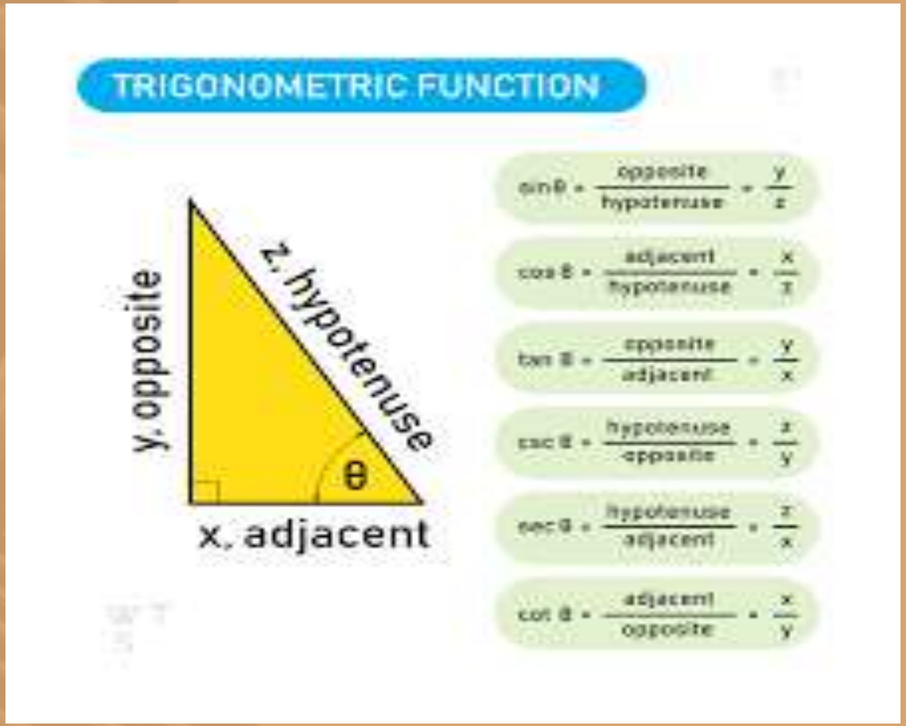
Trigonometric Functions

- ➢ The Main Identity : ***Sin*2** x + ***Cos*2** x = 1
- From this equations you can extract other equations

Trigonometric functions



Sin = opposite / hypotenuse, csc =1/sin
Cos = adjacent / hypotenuse, sec = 1/cos
Tan = opposite / adjacent, cot = 1/tan



Trigonometric identities

Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta}$$
$$\sec\theta = \frac{1}{\cos\theta}$$
$$\cot\theta = \frac{1}{\tan\theta}$$

Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1$$
$$\sec^2\theta = 1 + \tan^2\theta$$
$$\csc^2\theta = 1 + \cot^2\theta$$

Half-Angle Formulas

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$
$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$
$$\tan\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{\sin\theta}$$

Product to Sum Formulas

$$\sin x \sin y = \frac{1}{2}[\cos(x-y) - \cos(x+y)]$$
$$\cos x \cos y = \frac{1}{2}[\cos(x-y) + \cos(x+y)]$$
$$\sin x \cos y = \frac{1}{2}[\sin(x+y) + \sin(x-y)]$$
$$\cos x \sin y = \frac{1}{2}[\sin(x+y) - \sin(x-y)]$$

Sum to Product Formulas

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$$
$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

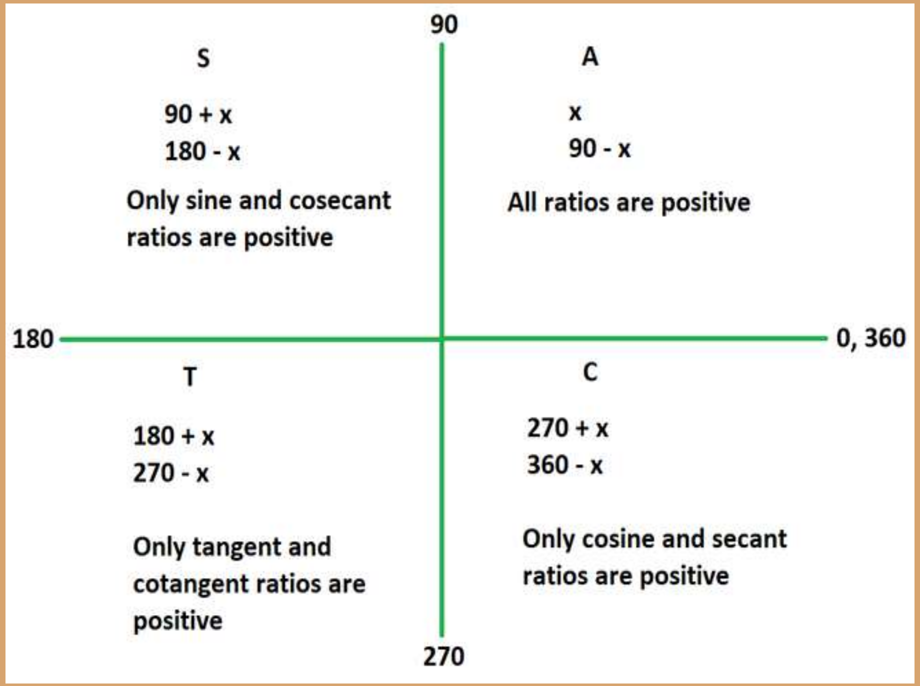
Coordinates

- As shown as in figure 1: the vertical line is called, Y-axis And the horizontal line is called X-axis

➢We use the coordinates to represents data

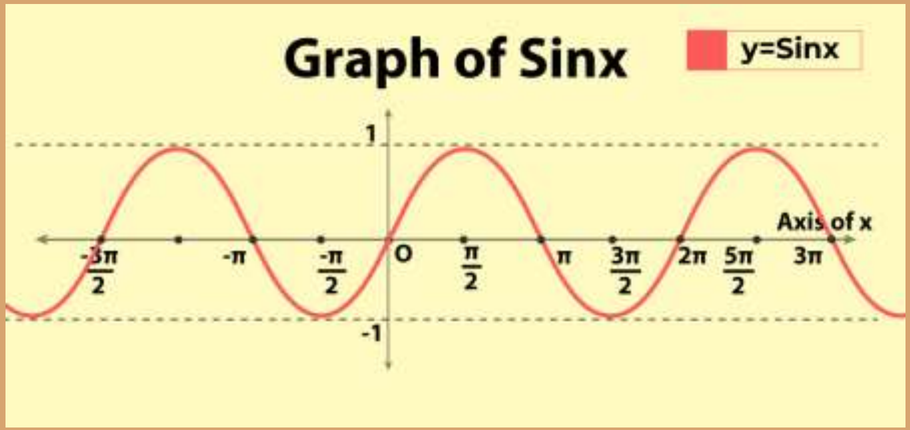
➢The first coordinate is divided to four parts are: the first coordinate, second coordinate, third coordinate and fourth coordinate

- the ordered pair in first coordinate is (+,+)
- the ordered pair in second coordinate is (-,+)
- the ordered pair in third coordinate is (-,-)
- the ordered pair in fourth coordinate is (+,-)



Graph of sine function

We use these graphs to represent equations and every equation is different from the other one



Generic equation: A sin (Bx + C)+D

A,B,C and D are variables you can change them, and the equation depended on these variables

I will give you some examples to elaborate what I mean

The equation: Y=sin x

This graph represent this equation

We find that A= 1 , B=0 , C=0 ,D=0

Another example

Y=2sin x

From this example we can indicate

The amplitude is increasing that mean

A (the number behind sin or cos)

Responsible for amplitude

Properties of the Sine Function $y = \sin x$

1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The sine function is an odd function, as the symmetry of the graph with respect to the origin indicates.
4. The sine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, \ldots$; the y-intercept is 0.
6. The maximum value is 1 and occurs at $x = \ldots, -\frac{3\pi}{2}, \frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}, \ldots$; the minimum value is -1 and occurs at $x = \ldots, -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{2}, \ldots$.

example

Y=sin(2 X)

We will find that this period end by not 2 π because we Depended on B(the number behind X)

And if we want to know the perid and taking B And use this quation. Period

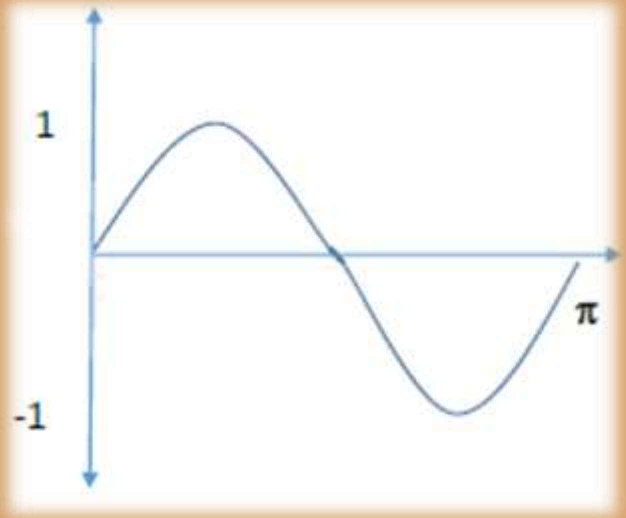
Period=$2\pi/B$ Y= 2 sin(4 X) 3

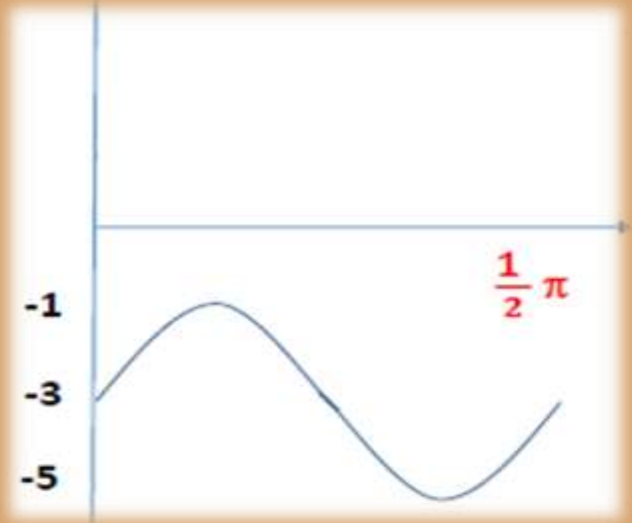
First of all, we want to find period

From the equation Period = $= \frac{2\pi}{B} = \frac{2\pi}{4} = \frac{1}{2}\pi$

If you find number after bracket and his Letter C you can back to the generic equation

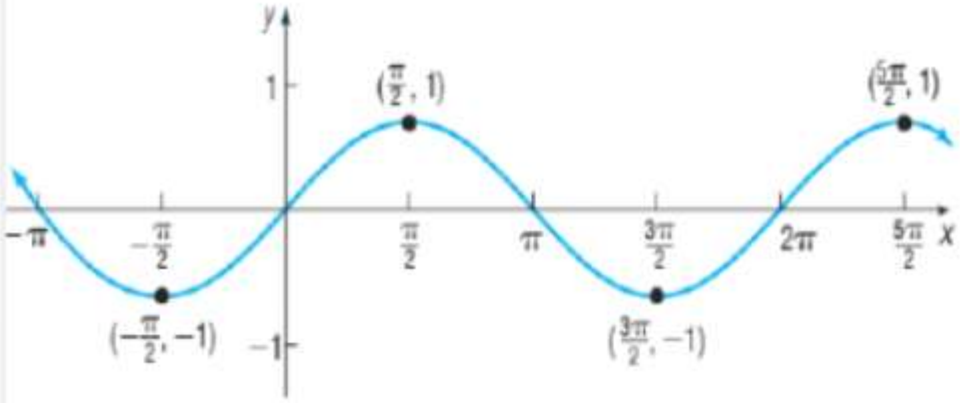
And find it. This number mean the graph Shift up or down if this sgin is negative as Shown as in the equation you will shift Down and if the sign is positive,you will Shift up



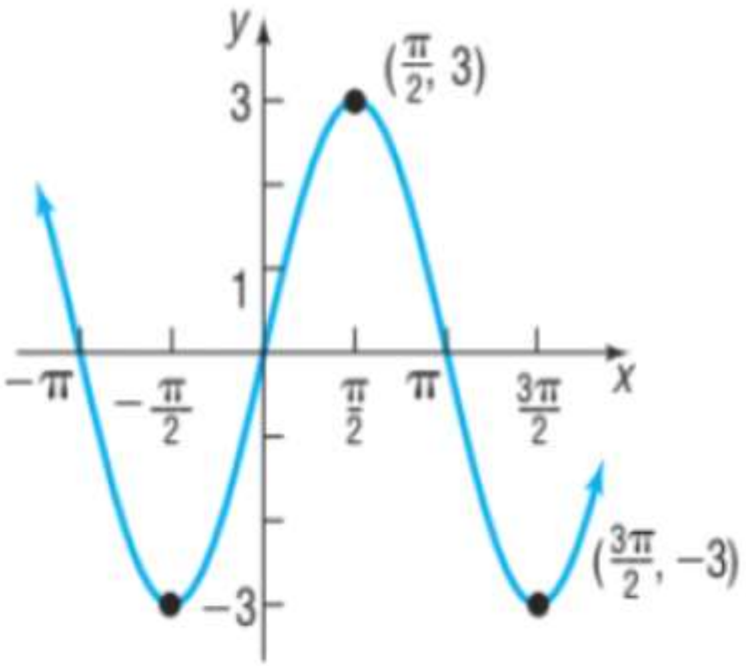


Graphing Functions of the Form $y = A\sin(\omega x)$ Using Transformations

Graph $y = 3\sin x$



(a) $y = \sin x$

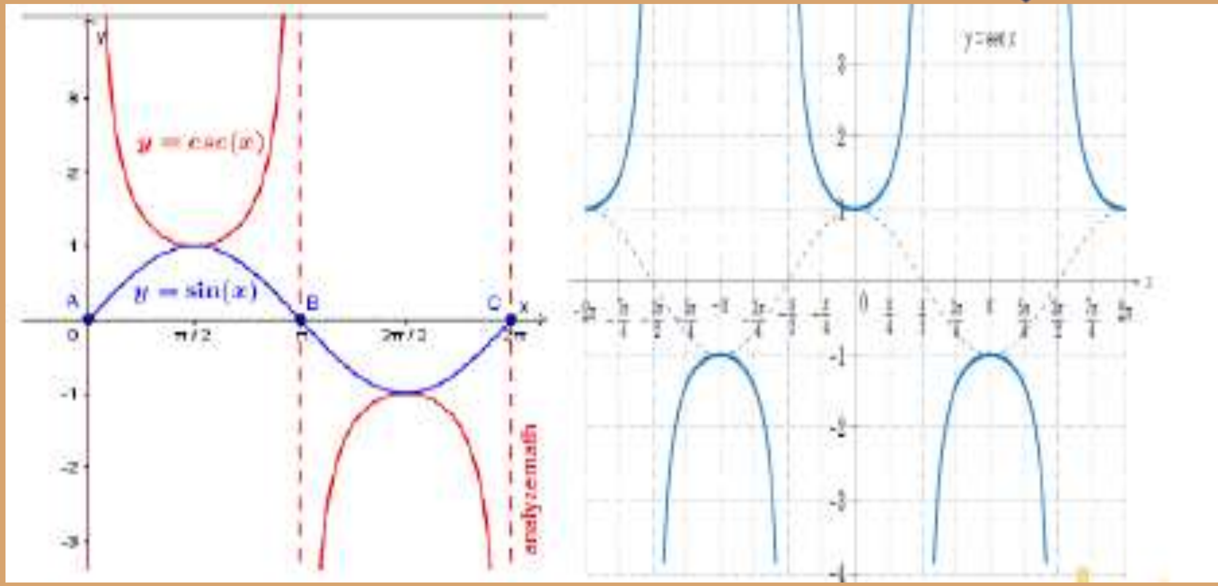


(b) $y = 3\sin x$

Graphs of sec, csc

- You can get these graphs from the sin and cos because there are flipped of there and theses line also asymptotes

- That is mean, these line or curved don t touch the verticals line because on these line sin X = 0 and csc is flipped of sin and you can't divide by zero the same thing in cos and sec

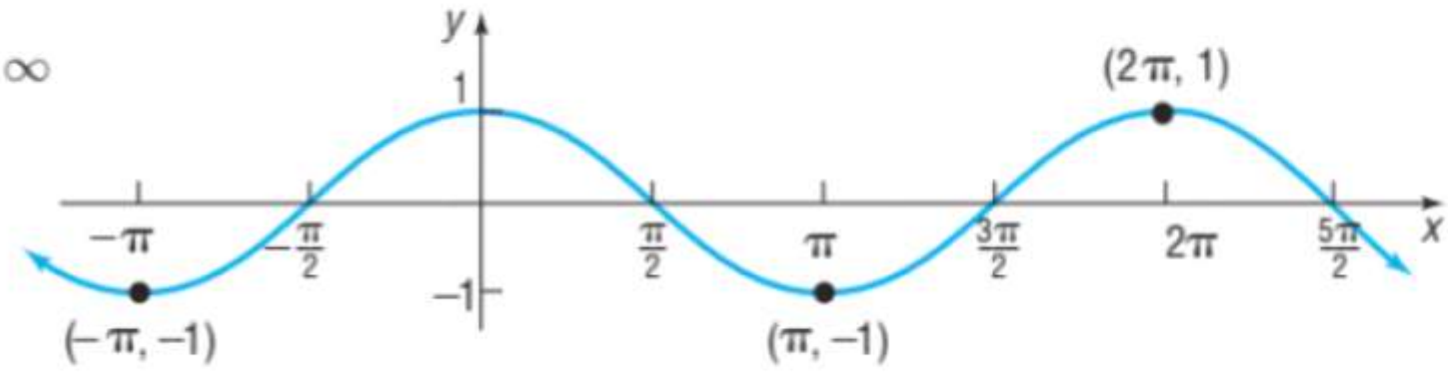


Graph of cosine function

The Graph of the Cosine Function

The cosine function also has period 2π. We proceed as we did with the sine function

$y = \cos x, -\infty < x < \infty$

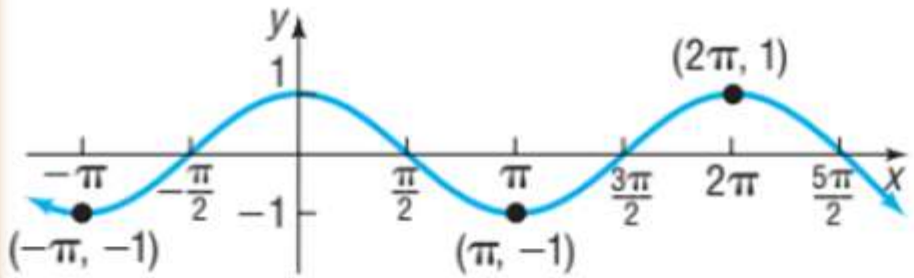


Properties of cosine function

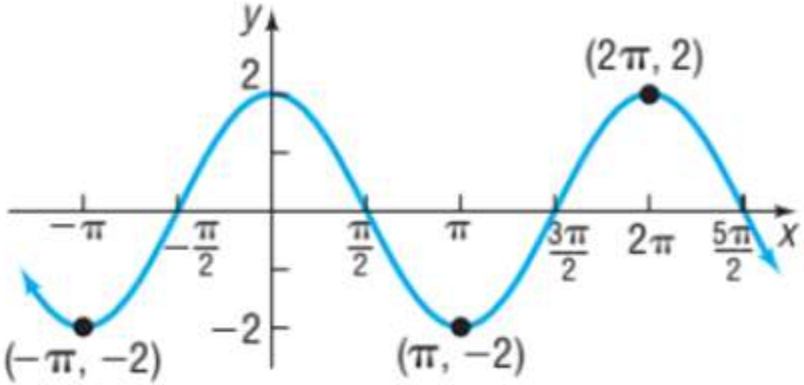
1. The domain is the set of all real numbers.
2. The range consists of all real numbers from -1 to 1, inclusive.
3. The cosine function is an even function, as the symmetry of the graph with respect to the y-axis indicates.
4. The cosine function is periodic, with period 2π.
5. The x-intercepts are $\ldots, -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$; the y-intercept is 1.
6. The maximum value is 1 and occurs at $x = \ldots, -2\pi, 0, 2\pi, 4\pi, 6\pi, \ldots$; the minimum value is -1 and occurs at $x = \ldots, -\pi, \pi, 3\pi, 5\pi, \ldots$.

Graph Functions of the Form $y = A\cos(\omega x)$ Using Transformations

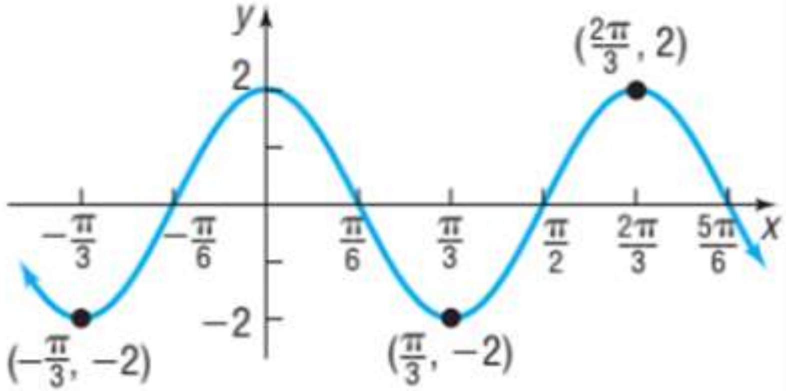
Graph $y = 2\cos(3x)$



Multiply by 2;
Vertical stretch
by a factor of 2



Replace x by 3x;
Horizontal
compression by
a factor of $\frac{1}{3}$



Inverse sine , cosine , and tangent function

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properties of a one-to-one function f and its inverse function f^{-1}

1. $f^{-1}(f(x)) = x$ for every x in the domain of f and $f(f^{-1}(x)) = x$ for every x in the domain of f^{-1}.
2. Domain of f = range of f^{-1} and range of f = domain of f^{-1}.
3. The graph of f and the graph of f^{-1} are reflections of one another about the line $y = x$.

Inverse sine function

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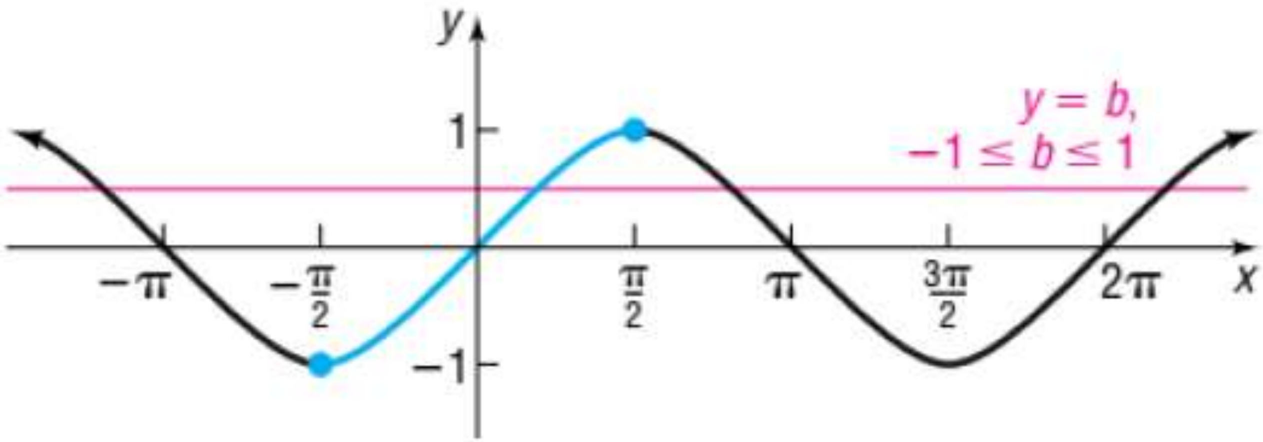
In Figure 1, we show the graph of $y = \sin x$. Because every horizontal line $y = b$, where b is between -1 and 1, inclusive, intersects the graph of $y = \sin x$ infinitely many times, it follows from the horizontal-line test that the function $y = \sin x$ is not one-to-one.

Figure 1

$y = \sin x, -\infty < x < \infty, -1 \le y \le 1$



How can define an inverse sine function on this restricted domain

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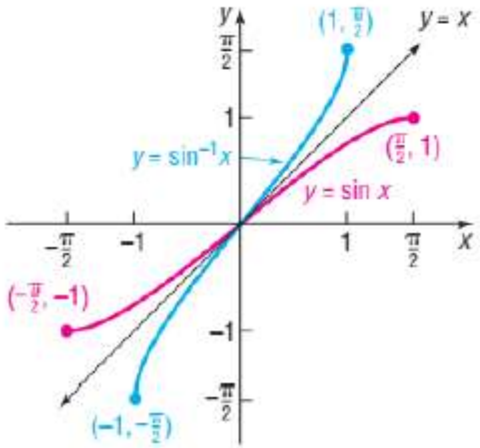
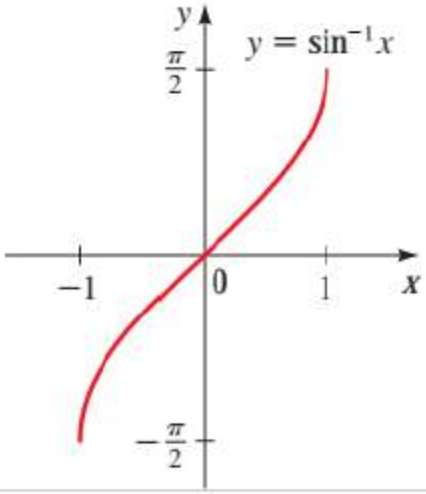
DEFINITION OF THE INVERSE SINE FUNCTION

The **inverse sine function** is the function $\sin^{-1}$ with domain $[-1, 1]$ and range $[-\pi/2, \pi/2]$ defined by

$$\sin^{-1} x = y \quad \Leftrightarrow \quad \sin y = x$$

The inverse sine function is also called **arcsine**, denoted by **arcsin**.





Domain and Range of Trigonometric Functions

Domain and Range of Trigonometric Functions are the values in which the function is defined and the output for the domain respectively. Trigonometric Ratios describe relationships between angles and the sides- of right triangles. A Function defined in terms of Trigonometric Ratios is called a Trigonometric Function. There are a total of six Trigonometric Functions. They play a significant role in various branches of mathematics and science, particularly in calculus and geometry. Trigonometric functions are essential in mathematics and have widespread applications in various fields.

What is Domain and Range?

Domain of a function is the set of input values (angles, in this case) for which the function is defined and produces a valid output. Trigonometric functions, such as sine, cosine, and tangent, have a domain that includes all real numbers. However, their range varies. **Range** of a function is the set of all possible output values. To calculate trigonometric values, you use the ratios. For example, $\sin(\theta)$ = opposite/hypotenuse, $\cos(\theta)$ = adjacent/hypotenuse, and $\tan(\theta)$ = opposite/adjacent.

Example:

Common trigonometric values include sin(30°) = 1/2, cos(45°) = √2/2, and tan(60°) = √3.

Sine and cosine functions have a range of [-1, 1], representing the amplitude of oscillation.

Tangent has a domain restriction excluding odd multiples of π/2 (-π/2, π/2, 3π/2, etc.), but its range is all real numbers.

Domain and Range of Sine Function {Sin(θ)}
The sine function sin(θ) has a domain of all real numbers (−∞, ∞) and a range between -1 and 1 inclusive: **$-1 \leq \sin(\theta) \leq 1$.**

Domain and Range of Cosine Function {Cos (θ)}
The cosine function cos(θ) also has a domain of all real numbers and a range between -1 and 1: **$-1 \leq \cos(\theta) \leq 1$.**

Domain and Range of Tangent Function {Tan (θ)}
The tangent function, tan(θ) has a domain of all real numbers except the values where the cosine is 0. So, its domain is (-∞, ∞) excluding the values where cos(θ) = 0 which are odd multiples of π/2. Its range is all real numbers covering the entire real number line.

Domain and Range of Cotangent Function {Cot (θ)}

The cotangent function cot(θ) shares its domain with the tangent function excluding the points where tan(θ) = 0 (i.e., multiples of π). Its range like tan(θ) covers the entire real number line.

Domain and Range of Secant Function {Sec (θ)}

The secant function, sec(θ) has a domain of all real numbers except the values where cos(θ) = 0. Its range is also (-∞, ∞) excluding values where cos(θ) = 0.

Domain and Range of Cosecant Function {Cosec (θ)}

The cosecant function, cosec(θ) shares its domain with the sine function, excluding the values where sin(θ) = 0 (i.e., multiples of π). Its range is (-∞, ∞) excluding values where sin(θ) = 0.

Trigonometric Function	Domain	Range
sin(θ)	R	[-1, 1]
cos(θ)	R	[-1, 1]
tan(θ)	R excluding odd multiples of π/2	R
cot(θ)	R excluding multiples of π	R
sec(θ)	R excluding values where cos(x) = 0	R-[-1, 1]
cosec(θ)	R excluding multiples of π	R-[-1, 1]

Domain and Range of Inverse Trigonometric Functions

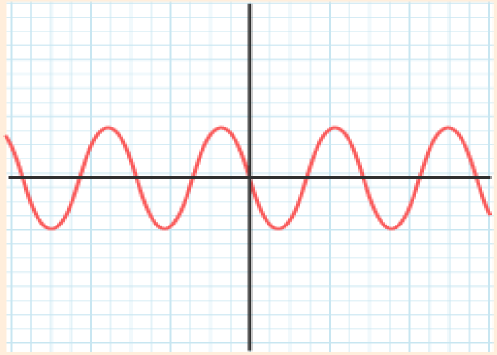
Inverse trigonometric functions are fundamental tools in calculus in the integration of trigonometric expressions. The behavior of inverse trigonometric functions helps in understanding the periodicity and symmetry inherent in trigonometric relationships.

The domain and range of inverse trigonometric functions such as $\sin^{-1}(x)$, $\cos^{-1}(x)$, $\tan^{-1}(x)$, $\cot^{-1}(x)$, $\sec^{-1}(x)$ and $\text{cosec}^{-1}(x)$ are specific to ensure one-to-one correspondence between inputs and outputs.

Inverse Trigonometric Function	Domain	Range
$\sin^{-1}x$	[-1, 1]	[-π/2, π/2]
$\cos^{-1}x$	[-1, 1]	[0, π]
$\tan^{-1}x$	R	(-π/2, π/2)
$\cot^{-1}x$	R	(0, π)
$\sec^{-1}x$	R – (-1, 1)	[0, π] – {π/2}
$\text{cosec}^{-1}x$	R – (-1, 1)	[-π/2, π/2] – {0}

Domain and Range of Trigonometric Functions Using Graph

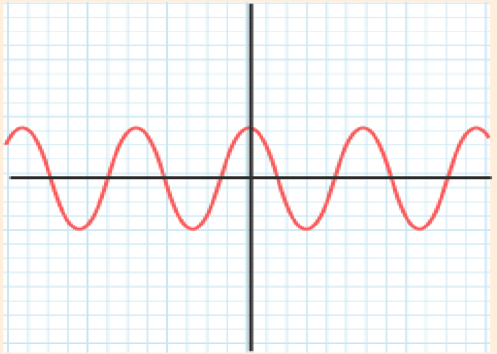
Sin θ



Domain = $(-\infty, +\infty)$

Range = $(-1, +1)$

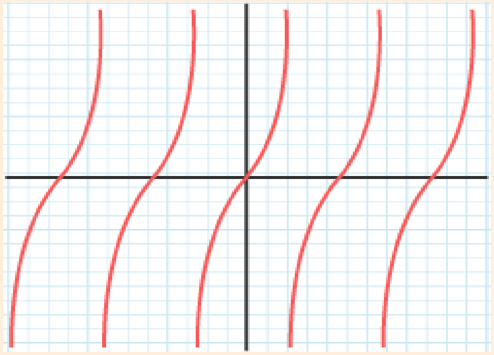
Cos θ



Domain = $(-\infty, +\infty)$

Range = $(-1, +1)$

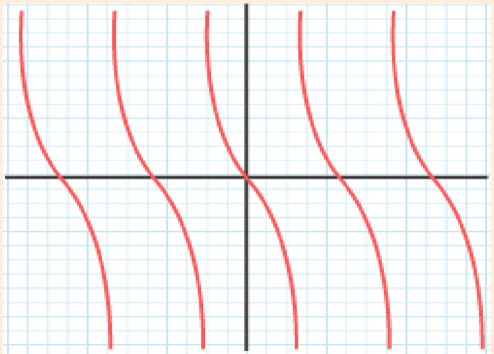
Tan θ



Domain = $R - \frac{(2n+1)\pi}{2}$

Range = $(-\infty, +\infty)$

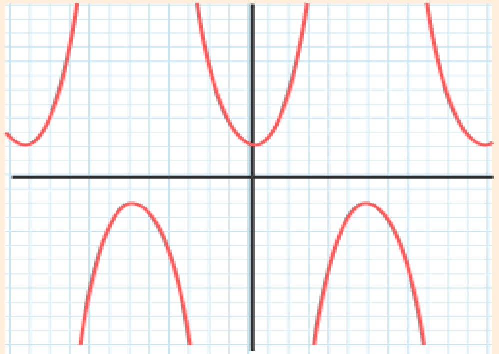
Cot θ



Domain = $(-\infty, +\infty)$

Range = $(-1, +1)$

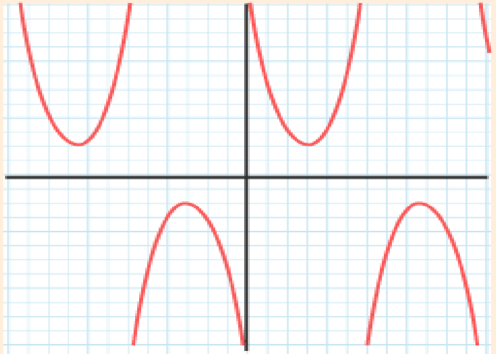
Sec θ



Domain = $R - \frac{(2n+1)\pi}{2}$

Range = $(-\infty, -1][+1, +\infty)$

Cosec θ



Domain = $R - n\pi$

Range = $(-\infty, -1][+1, +\infty)$

Examples on Domain and Range of Trig Functions

Example 1: Find the domain and range of $y = \sin(x)$ for all real numbers.

Solution:

The domain and range of y will be as follows:

Domain: R

Range: $[-1,1]$

Example 2: Determine the domain and range of $y = 2\cos(3x)$ for all real values of x.

Solution:

The domain and range of y will be as follows:

Domain: R

Range: $[-2,2]$

Example 3: Determine the value of $y = \cot(2x)$ for $x = 8\pi$

Solution:

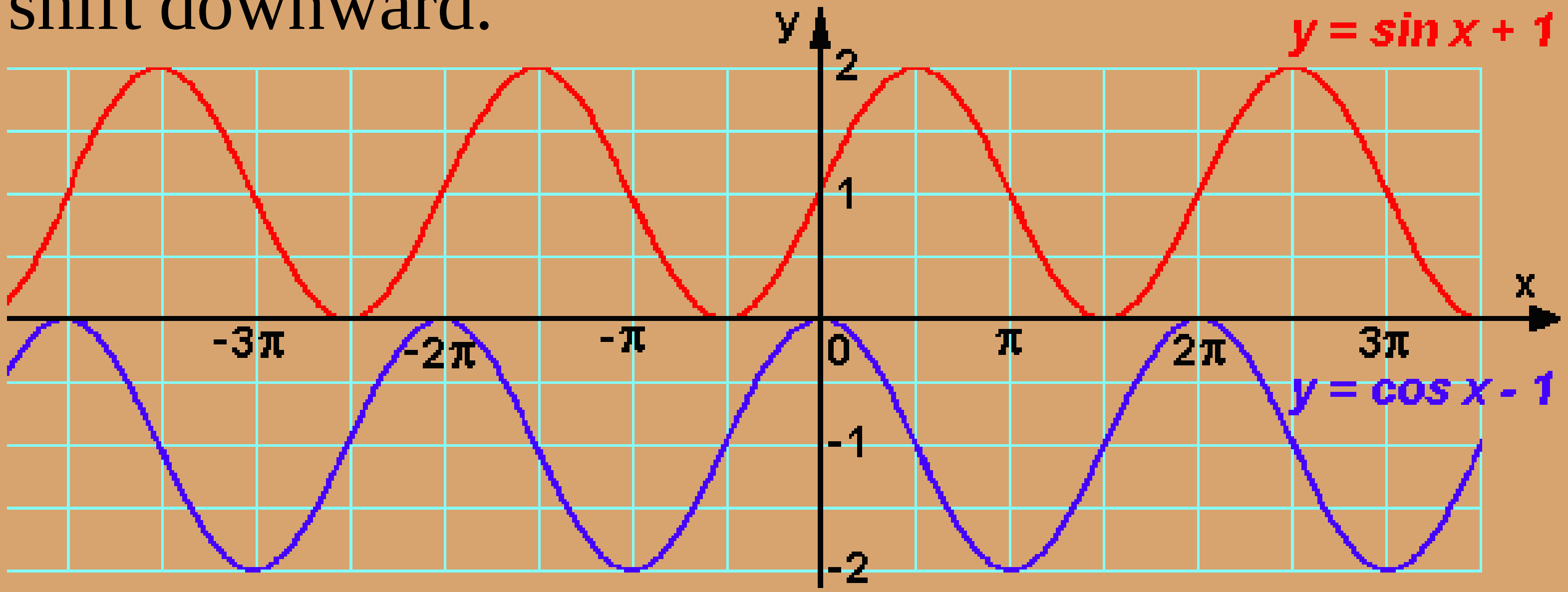
The value of y at $x= 8\pi$

$= \cot(2\times 8\pi)$

$= 1$

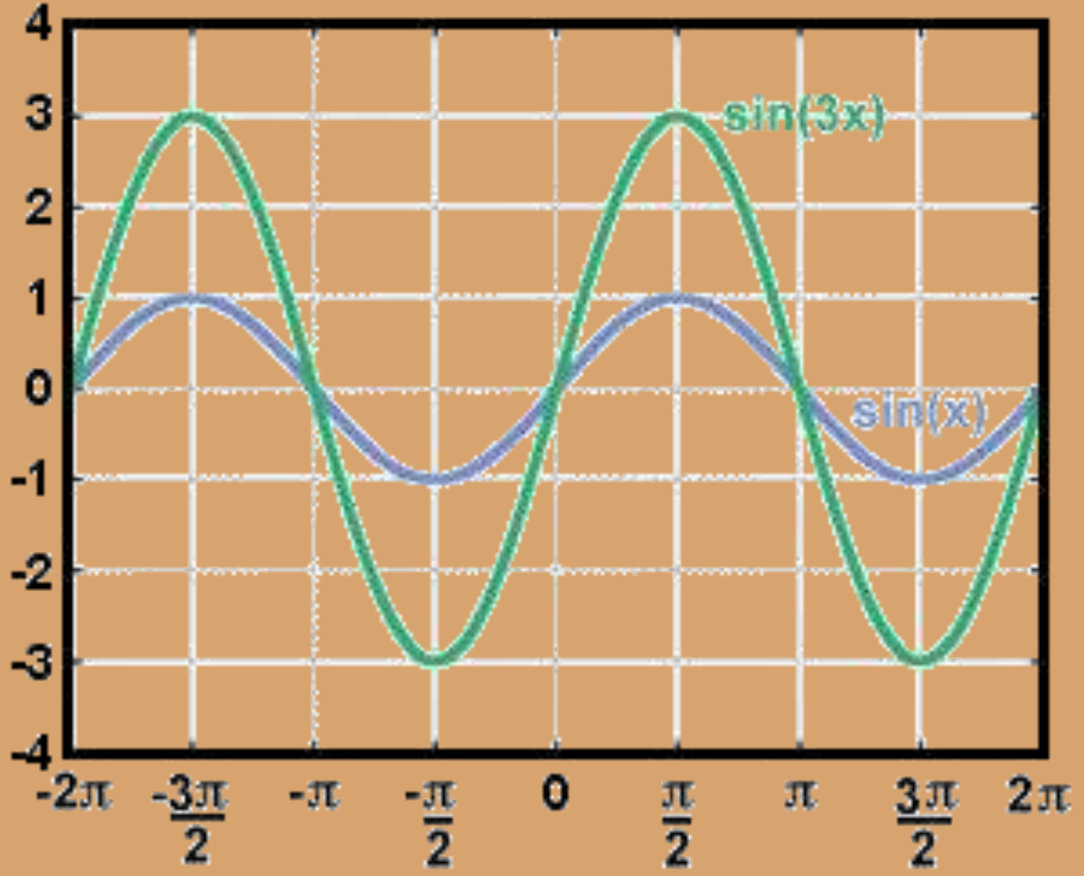
Vertical Shift of Sine and Cosine Functions

Adding a constant **B** to **sin** ***x*** or **cos** ***x*** in the equations of the function
y = sin *x* + B or **y = cos *x* + B**
causes a vertical translation of the graph.
For a positive **B** > 0 there is a shift upward.
For a negative **B** < 0 there is a shift downward.



Vertical Stretches

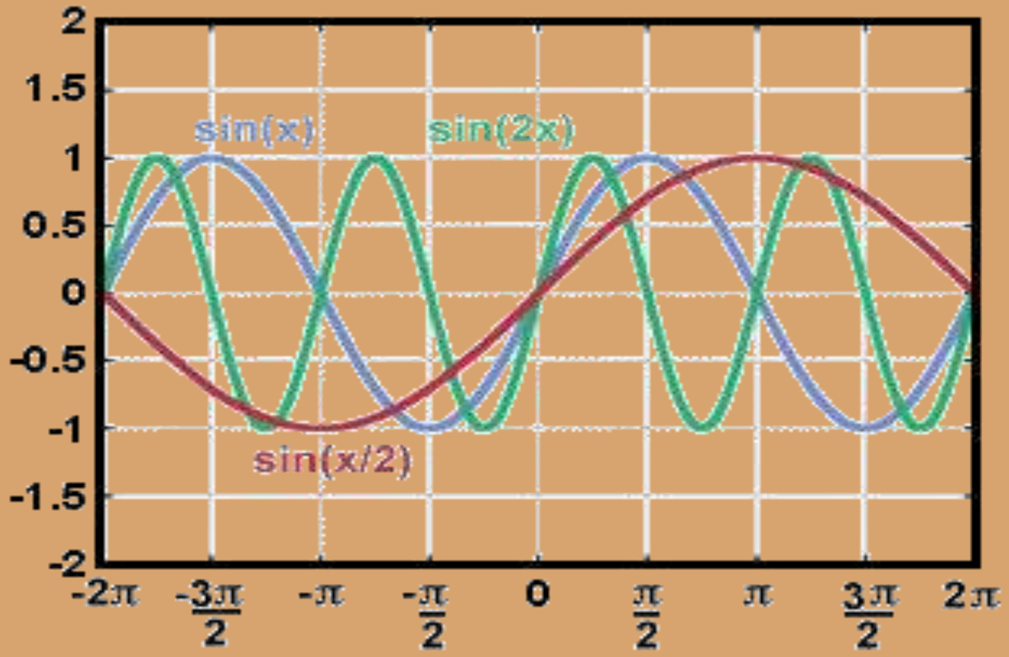
To stretch a graph vertically, place a coefficient in front of the function. This coefficient is the amplitude of the function. For example, the amplitude of $y = f(x) = \sin(x)$ is one. The amplitude of $y = f(x) = 3\sin(x)$ is three. Compare the two graphs below.



The amplitude of the graph of *any* periodic function is one-half the absolute value of the sum of the maximum and minimum values of the function.

Horizontal Stretches

To horizontally stretch the sine function by a factor of *c*, the function must be altered this way: $y = f(x) = \sin(cx)$. Such an alteration changes the period of the function. For example, continuing to use sine as our representative trigonometric function, the period of a sine function is , where *c* is the coefficient of the angle. Usually $c = 1$, so the period of the sine function is 2*Π*. Below are pictured the sine curve, along with the following functions, each a horizontal stretch of the sine curve: $y = f(x) = \sin(2x)$ and $y = f(x) = \sin($ $)$.



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